

## 3D ROCKING MOTION PREDICTABILITY: BLIND PREDICTION CONTEST RESULTS

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### Abstract

Rocking motion is sensitive to the boundary and initial conditions of a rocking structure. Thus, the claims that numerical rocking motion models are not only inaccurate, but that all rocking structures behave unpredictably. This paper revisits the issue of rocking motion unpredictability and explores whether not the response to a single ground motion, but the statistics of the responses to an ensemble of ground motions is predictable.

To this end, a 3D rocking podium structure was constructed and tested on a shake table. A blind prediction contest was organized, where the contestants were invited to predict the CDF of the time-maxima of the responses. There were 13 participant teams that used FEM, DEM and rigid body models.

It was found that several teams were able to predict the CDF with a decent accuracy. Both FEM and DEM can perform well, depending on the input parameters. Hence, it can be concluded that 3D rocking motion is predictable in the statistical sense.

**Keywords:** Rocking, Blind Prediction Contest, Shake table testing, Model Validation

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## 1 INTRODUCTION

Rocking structures are the ones that can uplift from their base when subject to an earthquake. Such structures attracted the attention of the research community after the powerful 1960 Valdivia (Chile) earthquakes, when Housner presented his seminal work [1] on the dynamics of the archetype rocking block (Figure 1). The rocking block dynamic model is useful because it can also describe the seismic behavior of non-anchored equipment [2-12], masonry structures [13-20], and ancient Greco-Roman and Chinese temples [20-22].

Motivated by the increasing stability of larger rocking blocks with identical aspect ratios, researchers have suggested using rocking as a seismic design strategy because the uplift of the block acts as a mechanical fuse and limits the design forces of both the superstructure and the foundation. The idea is applicable to both buildings [23-25] and bridges [26-40].

However, seismic analysis of rocking structures is not straightforward because, during the uplift, such structures have negative stiffness (in terms of the relation between the lateral force  $F$  and displacement  $u$ , shown in Figure 1). This is why their seismic response cannot be described by any “equivalent linear” system such as those employed for ordinary yielding structures [41]. Even though attempts to create non-linear spectrum-based methods to present the maxima of the seismic displacement of rocking structures (not related to elastic response spectrum) have been made [43-43], these spectra were created by performing multiple time-history seismic response analyses using Housner’s model. Therefore, Housner’s model is as useful for rocking structures as the elastic SDOF oscillator is for the fixed-base ones.

## 2 THREE-DIMENSIONAL ROCKING – “WOBBLING” UNDER 2D/3D GROUND EXCITATION

Housner’s model describes planar rocking of a rigid body excited by horizontal excitation. However, many real uplifting structures (e.g., bridges, statues, or ancient temples) would undergo a 3-dimensional motion characterized by simultaneous uplift from the ground (rocking) and change of the contact point with the ground (nutation) without twisting or sliding out of its original position, called “wobbling” in this study. Real uplifting structures may also twist and slide out of their positions, a phenomenon not investigated in this study.

There have been attempts to study wobbling using rigid body analytical dynamics. In [44], the motion of a rigid cylinder under seismic excitation was studied. Other researchers studied the 3D response of ancient conical or cylindrical columns [20, 45-48]. Makris et al. [49] experimentally tested scaled models of uplifting bridges that exhibited wobbling. All the above studies conclude that wobbling motion is present, even under planar initial conditions and/or

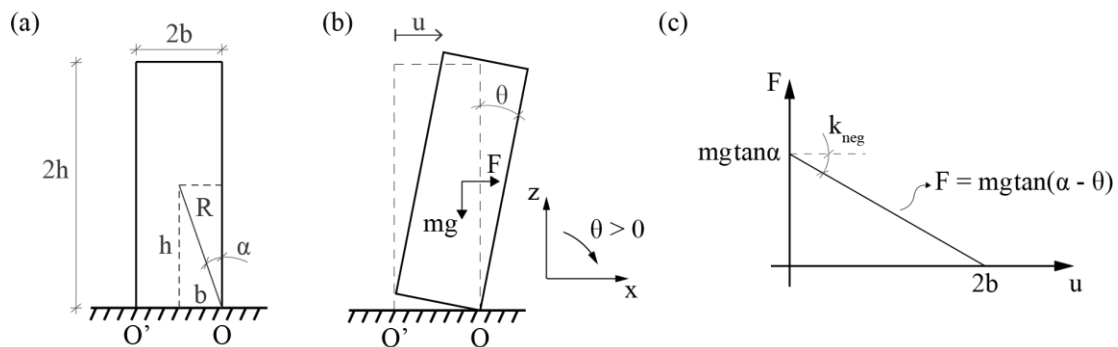


Figure 1: Geometric characteristics of a rigid rocking body (a,b); Lateral force-deformation relation of an uplifted rigid rocking body (c).

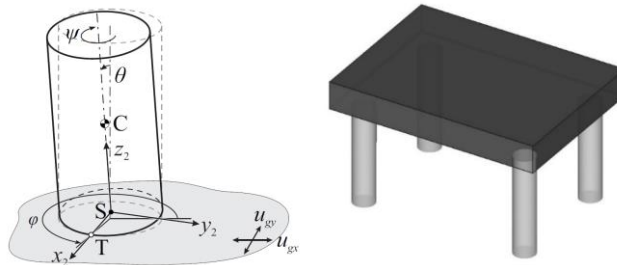


Figure 2: Cylinder allowed to wobble without twist [56] (Left); Wobbling frame [68] (Right).

under planar ground excitation. Stefanou et al. [50] provided theoretical justification for the above observation.

The 3D behavior of non-cylindrical rocking bodies received attention recently. Konstantinidis and Makris [51] and Zulli et al. [52] studied the rocking motion of a 3D prism. Chatzis and Smyth [53] studied the motion of a 3D prism on a deformable base, taking sliding into account as well as the 3D dynamics of a rigid body with wheels on a moving base [54]. Mathey et al. [55] studied the influence of geometric defects on the 3D response of small-size blocks. They concluded that the blocks with imperfections are less stable than the theoretically perfect ones.

Vassiliou et al. [56] studied a 2DOF rigid cylinder constrained to wobble above its initial position without twisting or sliding, i.e. without stepping out of its original position (Figure 2, left). Subsequently, Vassiliou [57] extended this model to include a slab on top of a set of cylindrical columns (Figure 2, right). This is a model of a wobbling bridge, where the columns are physically constrained to wobble above their initial position without stepping out, under the assumption that wobbling is smooth and sliding on the ground does not occur.

### 3 STATISTICAL SEISMIC RESPONSE MODEL VALIDATION

A major drawback for rocking seismic response models is that they are not validated - at least not by means of the conventional approach of deterministically comparing the displacement response of the model and the prototype to a given ground excitation. This is further hindered by the fact that the seismic response of a rocking block is particularly sensitive to all of the parameters that define it and as such it has been characterized as “chaotic” (i.e. non-reproducible and non-predictable). In fact, small model parameter perturbations lead to substantially different time-history responses. For this reason, experiments involving seismically or dynamically excited rocking specimens are seldom repeatable.

This lack of validation effectively means that Housner’s model should not be used as its ability to represent the physical reality has not been proven. Consequently, rocking cannot be used as a seismic design strategy, given that the seismic response of such structures would be effectively unpredictable, by analytical or numerical methods.

Bachmann et al. [58, 59] and Del Giudice et al. [60] suggested that the conventional seismic response model validation procedure focused on deterministic reproduction of the experimentally obtained response to a particular ground motion with acceptable accuracy is too strict of a test. Instead, they proposed a weaker, but sufficient, concept of statistical validation of structural seismic response models. This novel model validation procedure comprises two steps. *First*, an experimental benchmark dataset, measuring the recorded dynamic response of the same specimen (or essentially identical specimens) subjected to ensembles of consistently generated and scaled ground motions that represent a given seismic hazard, is developed. *Sec-*

ond, a model is used to produce a dataset comprising its dynamic responses to the same ground motion ensembles. Validation is conducted by comparing statistical distributions of the pertinent response quantities of the model and the benchmark using the two datasets. Statistical seismic response model validation is a weak method in that it involves the statistics of model predictions of the response to ensembles of ground motions, rather than predicting the responses to every single ground motion deterministically. Bachmann et al. [58], [59] applied the statistical seismic response model validation procedure to validate the Housner [1] rocking response model. They performed 600 shaking table tests using a well-defined and repeatable uplifting structure as well as 600 numerical simulations for the same tests, and compared both the individual test responses and the statistical aggregates of these responses focused on predicting limit states such as overturning or maximum tilt angle. They showed that the 1963 Housner model passes the weak validation test, even though it fails the strong validation test. Therefore, the Housner model was found to be good enough for the use in the scope of seismic design.

#### 4 WOBBLING SEISMIC RESPONSE BENCHMARK DATASET

Wobbling (three-dimensional rocking without sliding) is even harder to predict than planar rocking. Thus, developing seismic wobbling models is challenging, but is necessary in order to further develop seismic design of rocking structures as well as of non-structural elements, such as equipment, as outlined in the introduction of this paper. Validation of wobbling models is a key step in this development process. To facilitate statistical seismic response model validation of wobbling response models, a series of shake table tests was performed at the University of Bristol to create the benchmark response dataset.

##### 4.1 Specimen Description

The wobbling specimen was designed at ETH Zurich and built in Bristol in the framework of the EU-funded research project SERA [61,62]. It comprises an aluminum slab supported on four wobbling circular structural steel columns resting on the shake table platform (Figure 3). The four columns had a height, diameter and wall thickness of 1000 mm, 244.5 mm and 8 mm, respectively, with a corresponding slenderness (i.e. diameter to height) ratio of  $\tan\alpha = 0.2445$ . Note that the size of the wobbling specimen is not representative of the size of a prototype bridge or a prototype podium building. The ground motion excitations used in the



Figure 3: Wobbling specimen on the University of Bristol shake table.

shake table experiments were scaled to preserve acceleration scaling (as discussed later in this paper) so that the model represents a structure with a height similar to an actual bridge. The intent of the specimen detailing was to mimic the assumptions in Vassiliou's [57] "wobbling bridge" model, namely that the structure is rigid and that the columns wobble without sliding or twisting about their longitudinal axis. Conical end restraints were placed at each end of each column to stop the columns from wobbling out of their original position.

#### **4.2 Specimen Excitation Using A Shake Table**

The wobbling response of the specimen was induced by a dynamic bi-directional (two orthogonal horizontal components) excitation of its support. This was achieved by placing the specimen on the top of the 6DOF 3 m×3 m shake table of the University of Bristol [63].

#### **4.3 Ground Motions and Scaling**

To constrain the uncertainty in the ground motion excitation, ensembles of bi-directional ground motions were synthesized using a spectral version of the Rezaeian and Der Kiureghian stochastic ground motion model [64-66]). More info on the ground motions used can be found in [61].

Two recorded ground motions, the 1940 El Centro Array #9 record and the 1999 Chi-Chi CHY080 record, were used as the "seed" ground motions for the experimental campaign. Next, two bi-directional (two orthogonal horizontal components) ground motion ensembles, each comprising 100 synthetic ground motions, were generated from the two seed ground motions. These two ground motion ensembles were used to drive the shake table during the conducted tests. Vertical ground motion translational and the three rotational excitations were not considered.

A detailed discussion on the scaling parameters of the tests can be found in [61]: The frequency of the excitations was scaled by a factor of  $\sqrt{9.67}=3.11$ , so that the size of the structure in the prototype scale is 9.67 times larger and its height is 9.67m and representative of a typical overpass bridge.

It should be noted that the wobbling specimen is only a distorted model of a prototype structure because the stress similitude is not preserved. Thus, the elastic modulus of the material and the natural frequencies of the prototype are not correctly scaled. In addition, the wobbling specimen columns are not perfectly rigid, as Vassiliou [57] assumes, nor are any realistic foundation condition modelled. However, it has been shown that the deformability of large structures does not qualitatively change their rocking behavior [67-74]. Ultimately, the conducted experiments serve to generate a benchmark dataset for wobbling seismic response model validation and do not aim at representing the nuances of a prototype structure without distortion.

#### **4.4 Motion Measurement**

The response of the wobbling specimen was measured using displacement and acceleration sensors. The displacements were measured using an infrared tracking system, with six infrared passive markers installed on the top of the aluminum slab, as shown in Figure 4. The accelerations were measured using three-dimensional accelerometers. These sensors were placed on the shake table platform to record the actually applied excitation.



Figure 4: Marker positions on top of the aluminum slab used for wobbling specimen displacement measurements.

#### 4.5 Relevant Wobbling Response Quantity

The horizontal displacement of the slab is the relevant response quantity in the created benchmark dataset, which was measured using the markers shown in Fig. 4. The following maximum absolute average displacement (MAAD) of the wobbling specimen slab is defined as the relevant measure of the wobbling specimen response:

$$u = \max_t \left( \frac{u_1(t) + u_3(t) + u_4(t) + u_6(t)}{4} \right) \quad (1)$$

where  $u_i$  is the Euclidean norm of the horizontal displacement at points  $M_i$  of Fig. 4, relative to the shake table. Fig. 5 plots the Cumulative Distribution Functions (CDFs) of the specimen MAAD (i.e. of  $u$ ) for the El Centro and the Chi-Chi ground motion ensembles using thick black lines.

### 5 WOBBLING SEISMIC RESPONSE BLIND PREDICTION CONTEST

Statistical seismic response model validation was conducted for a number of wobbling response models in the scope of a blind prediction contest organized by the Pacific Earthquake Engineering Research center (PEER), the University of Bristol, and ETH Zurich, launched in October 2019 with a prediction submission deadline on November 20<sup>th</sup>, 2019. The contestants were asked to use their models to predict not the wobbling specimen seismic displacement (Equation 1) to each of the 200 ground motions in the two ground motion ensembles, but the experimental CDFs of the wobbling specimen MAAD  $u$  response, as shown in Fig. 5 using thick black lines. For each set of ground motion ensembles, the competing models were ranked according to the maximum vertical distance, i.e. the Kolmogorov-Smirnov (K-S) distance [75], between their predicted and the experimentally obtained CDFs. Given that two ensembles of ground motions were used, the average Kolmogorov-Smirnov distance was taken as a best-fit performance indicator. The as-built geometry of the wobbling specimen and the recorded shake table accelerations were provided to the contestants, but no tests were performed to mechanically characterize the materials of the wobbling specimen (i.e., the steel of the columns and conical restrainers), as these were not essential in terms of affecting the response and use of common modulus of elasticity for steel and aluminum was sufficient.

Thirteen contestants participated in the blind prediction contest. The models they used can be grouped into three categories, as follows:

- a) 8 contestants developed models based on the Finite Element Method (FEM), out of which 4 used ABAQUS [76] (Teams #7,#9,#10, and #12), 2 used SAP2000 [77] (Teams #2 and #4), 1 used Midas GEN [78] (Team #1) and 1 used OpenSees [79] (Team #6);



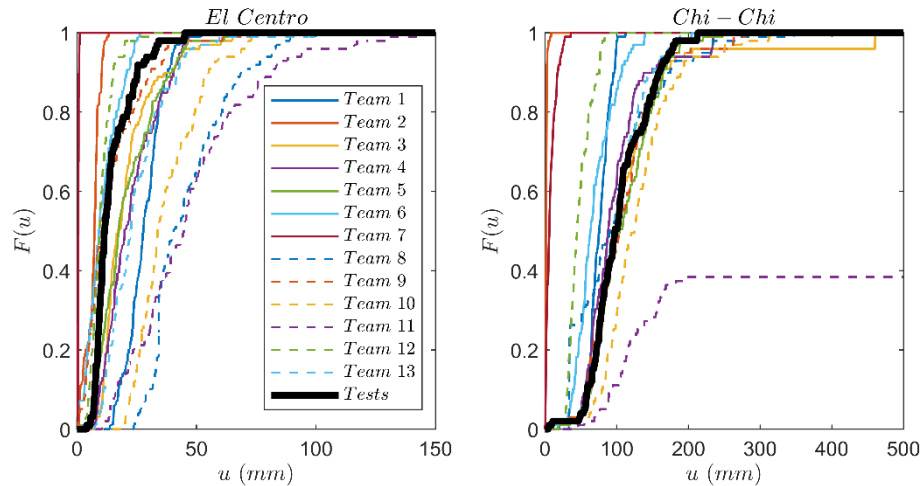


Figure 5: MAAD CDFs obtained from shake table tests (thick black lines) and CDFs of the same response quantity obtained using the 13 contesting models for the El Centro and Chi-Chi ground motion examples.

|    | El Centro                 |              | Chi-Chi                   |              | Average                   |              |
|----|---------------------------|--------------|---------------------------|--------------|---------------------------|--------------|
|    | Team number (method used) | K-S Distance | Team number (method used) | K-S Distance | Team number (method used) | K-S Distance |
| 1  | 12 (FEM)                  | 0.25         | 3 (DEM)                   | 0.06         | 9 (FEM)                   | 0.19         |
| 2  | 9 (FEM)                   | 0.27         | 9 (FEM)                   | 0.11         | 3 (DEM)                   | 0.21         |
| 3  | 6 (FEM)                   | 0.32         | 13 (DEM)                  | 0.11         | 5 (DEM)                   | 0.24         |
| 4  | 5 (DEM)                   | 0.34         | 5 (DEM)                   | 0.13         | 4 (FEM)                   | 0.30         |
| 5  | 3 (DEM)                   | 0.36         | 4 (FEM)                   | 0.16         | 13 (DEM)                  | 0.30         |
| 6  | 4 (FEM)                   | 0.43         | 8 (DEM)                   | 0.26         | 6 (FEM)                   | 0.39         |
| 7  | 13 (DEM)                  | 0.48         | 10 (FEM)                  | 0.27         | 12 (FEM)                  | 0.48         |
| 8  | 2 (FEM)                   | 0.65         | 6 (FEM)                   | 0.46         | 10 (FEM)                  | 0.53         |
| 9  | 1 (FEM)                   | 0.69         | 1 (FEM)                   | 0.48         | 8 (DEM)                   | 0.58         |
| 10 | 11 (RB)                   | 0.72         | 11 (RB)                   | 0.62         | 1 (FEM)                   | 0.59         |
| 11 | 10 (FEM)                  | 0.79         | 12 (FEM)                  | 0.71         | 11 (RB)                   | 0.67         |
| 12 | 8 (DEM)                   | 0.89         | 7 (FEM)                   | 0.98         | 2 (FEM)                   | 0.82         |
| 13 | 7 (FEM)                   | 1.00         | 2 (DEM)                   | 0.99         | 7 (FEM)                   | 0.99         |

Table 1. Contestant model ranking based on the Kolmogorov-Smirnov distance between the MAAD CDFs

- b) 4 contestants used the Discrete Element Method (DEM) to build their models, out of which 3 used 3DEC [80] (Teams #3, #5 and #8), and 1 used Code Aster [81] (Team #13);
- c) 1 contestant (#Team 11) used Vassiliou's [57] Rigid Body (RB) model (independently, as Vassiliou was one of the contest organizers and did not take part in the contest).

The outcomes of the statistical seismic response validation of the contesting models are shown in Figure 5 and Table 1. Figure 5 graphically compares the predicted wobbling specimen MAAD response CDFs ( $F(u)$ ) to the experimentally obtained wobbling specimen dis-

placement response CDFs. Table 1 ranks the contestant predictions based on the K-S distance: Column 1 gives the ranking for the El Centro ensemble, column 2 for Chi-Chi ensemble, and column 3 gives the overall model ranking based on the average Kolmogorov-Smirnov distance for the two ensembles of ground motions.

Based on the K-S distance, the winning model is the one submitted by Team #9 (Myron Chiyun Zhong and Constantin Christopoulos from the University of Toronto). They used FE software ABAQUS 6.13 with C3D20R quadratic brick element, with reduced integration. They assumed a coefficient of friction of 0.3 and no Rayleigh damping [82]. The second best winning model was the one submitted by Team #3 (Daniele Malomo, Anjali Mehrotra, and Matthew DeJong from UC Berkeley). They used Distinct (or Discrete) element software 3DEC. The frictional mechanisms were modeled using a simplified Mohr-Coulomb criterion with tension cut-off [83]. Zero Rayleigh damping was used.

The blind prediction contest provided a unique opportunity to conduct simultaneous statistical seismic response model validation for 13 different models. Ranking of the models depends on the ground motion ensemble characteristics. Generalized conclusions about which modelling methods and parameters are better for which ground motions cannot be drawn because only two ensembles were tested. However, the models of Teams #6 and #12 deserve special attention as they performed notably better in the El Centro ensemble than in the Chi-Chi one. The first comment that holds for both Team #6 and Team #12 models is related to the direction of the errors in the El Centro ensemble. The sign of the errors (obscured by the MAAD response quantity) indicates that both models approach the experimental benchmark MAAD CDF of the El Centro ensemble “from the left” (Figure 5), i.e. they underestimate the response. As the rocking podium has a pronounced softening behavior, it is expected that such underestimation of wobbling specimen displacements will only increase for ground motions in the more intense Chi-Chi ensemble. Therefore, models that underestimate the response for low-intensity ground motions are generally expected to underestimate the response even more in more intense ground motion response simulations, which is exactly what is observed from the predictions of Teams #6 and #12 for the Chi-Chi ensemble. Why do the models of Teams #6 and #12 underestimate the response in the first place? Team #6 used OpenSees and modelled the rocking surface with two independent rotational springs, meaning that the model responds independently in the two orthogonal directions. Neglecting such interaction has already been proven to underestimate the response ([71]). Not enough model details were provided to understand why model #12 underestimated the response.

Both FEM and DEM models used in this blind prediction contest performed well or poorly, depending on the modeling parameters. For example, of the two best models, Team #9 developed an FEM and Team #3 used a DEM model. Thus, there is no basis to recommend FEM or DEM to model wobbling structures seismic response. However, it should be mentioned that MAAD CDFs produced by DEM models were close to the experimental benchmark CDFs, except in the case of the model developed by Team #8 which clearly underpredicted the response for the El Centro ground motion ensemble. Similarly, the FEM models of Teams #2 and #7 grossly underestimated the response, as can be seen in Figure 5. This does not mean that FEM models cannot perform well, or that such models would underestimate the response; the FEM model of Team #9 performed the best in all metrics. This is merely an indication that, when it comes to uplifting and rocking structures, modeling decisions should be made with care. This is more important for FEM models, as such models originate from continua, as opposed to DEM models that inherently assume the structure comprises of discrete parts.

The specific Rigid Block model proposed by Vassiliou [57] and used by Team #11 consistently overestimates the response. Hence it is conservative, but not accurate. It should be noted that the Rigid Block model formulation does not include any form of energy dissipation.



Apparently, disregarding the energy dissipated in the tests through scratching of the conical restrainers leads to overestimation of the response. Therefore this model can only be used to obtain conservative estimates of the response.

## 6 CONCLUSIONS

The seismic response of rocking structures is notoriously sensitive to all of the parameters that define it. As expected, numerical models often fail to adequately predict the seismic response of a rocking structure to a single ground motion. Based on this, researchers have claimed that not only are current models inaccurate, but that a numerical model accurate enough to predict the seismic response of rocking structures is not feasible. Hence, rocking is not used as a seismic design approach, and virtually all engineered structures are designed as fixed to the ground.

Along these lines, this paper revisits the issue of seismic response predictability in seismic design. It claims that predicting the response to a single ground motion is too strong of an acceptance test for a model. Instead, the minimum precondition for a valid model to be used in seismic design is the ability to predict the statistical characteristics of the seismic responses to an ensemble of ground motions that are compatible to the seismic hazard of interest. This is a weaker, but sufficient model validation test.

The objective of this study is to answer the question: Are seismic tests of three-dimensional rocking structures predictable using numerical models? To answer the above question in a statistical sense, an experimental campaign has been designed to obtain observations of the seismic response of a rocking podium structure that was able to sustain three-dimensional rocking motion without sliding. A stochastic model was used to generate two synthetic ground motion ensembles that match the physical characteristics of two recorded ground motions. These two ensembles were used to excite a rocking podium structure.

A blind prediction contest was organized to evaluate the ability of several models submitted by the contestants to predict the response of the rocking podium structure statistically. The models were evaluated based on their ability to predict not the time history response to each single ground motion, but the CDF of the maxima of the specimen displacement response time histories. Contestants used FEM, DEM and Rigid Body Dynamics models.

The winning model, developed by means of the FEM, scored the best of all models. However, this is not sufficient to conclude that FEM models were uniformly better than the submitted DEM models, because some DEM submissions were also very accurate, plus some of the least accurate models were FEM. Therefore, both FEM and DEM can perform well or poorly in simulating the wobbling (three-dimensional rocking) seismic response, depending on the parameters used and the assumptions made. It was also found that the numerical models were able to predict better the specimen displacement CDF obtained using the Chi-Chi ground motion ensemble than the El Centro one. The Chi-Chi ensemble is more intense and induced larger specimen displacements, which is an indication that the numerical models predict larger wobbling displacements better than small ones, hence they are expected to do well in predicting overturning. Therefore, it can be concluded that the response of a 3-dimensional rocking structure is predictable in terms of a CDF of the maxima of the time-history responses to a set of ground motions.

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